Reviver: A Practical Provable Surface Reconstructor

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Abstract

In this paper we describe REVIVER [1] a surface reconstructor that takes unorganized points as input and outputs a piecewise linear surface interpolating the input point set. The algorithm used in the design of *Reviver* is simple, fast and provable. It first constructs a Delaunay triangulation of the input and then extracts the surface provably when the sampling is dense. The theoretical framework provided by the pioneering work in surface reconstruction by Amenta and Bern[1] is used in the design of *Reviver* 's algorithm. The most significant achievement of *Reviver* is the speed with which it reconstructs!

1 Introduction

Fast and provable surface reconstruction from unorganized point sets can always find applications in varied areas. Unfortunately, most current implementations for reconstruction in three dimension come without theoretical guarantees and provable reconstruction algorithms are too slow to be useful in real life. In this paper we describe the design of REVIVER a reconstructor of unorganized three dimensional points that is both provable and fast. *Reviver* is only a step towards the speed needed to reconstruct millions of points using provable algorithms.

Reviver is based upon the theoretical framework provided by the pioneering work by Amenta and Bern[1]. The algorithm of *Reviver* extracts the interpolating surface directly from the Delaunay tetrahedralization of the input points without either resorting to the computation of the Voronoi Diagram or the insertion of more points(poles) in the Delaunay tetrahedralization. *Reviver* works by just walking from one 'good' triangle to the other in the tetrahedralization. When this walk is complete, it is guaranteed that the reconstruction is 'correct' provided that the input points are a dense enough sample of a smooth surface.

Figure 1. A Point set with 55k points reconstructed by Reviver in less than a minute. This data set does have "sharp edges" and "borders".

Reviver can reconstruct 10,000 points in a few seconds on a P-II 350. Most of the time spent in *Reviver* is taken in doing Delaunay triangulation using *Qhull*. The speed at which Qhull works on a machine is the bottleneck for any machine to use *Reviver* . This means that on modern machines like a 1Ghz pentium one could reconstruct a 100000 point set[2] using reviver in a few seconds. It also implements an $\alpha-$based filtering algorithm to decimate delaunay triangles based upon the Alpha shape [3] of the input point set. This step is done prior to the walk on the triangles of the Delaunay tetrahedralization and hence the complexity of the walk can be reduced. By applying a α-filter, the speed of *Reviver* can be further increased. *Reviver* uses the LEDA library[3] for data structures and Qhull for delaunay triangulation.

[1] This research is supported by the National Science Foundation (CCR-9732220 and CCR-0098172) and from Sandia National Labs. REVIVER was released for beta testing more than a year ago!! All timings/figures reported are from the Beta version running on P-II 350Mhz/256Mb RAM.

[2] On my PIII 1Ghz laptop with 512Mb RAM I could triangulate a 100000 points sampled on a sphere in 13 seconds!

[3] LEDA version 3.8 was used, the main use being Basic data structures like Set, Lists and Exact Computation after hand crafted floating point filters if required. I couldnt yet make LEDA 4.3 workable on my laptop, to report *Reviver* timings on a P-III 1Ghz.

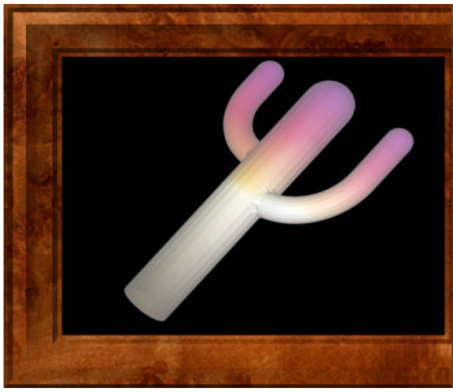

Figure 2. Cactus, 3337 points in 25 sec, Input with sharp turns and border edges.

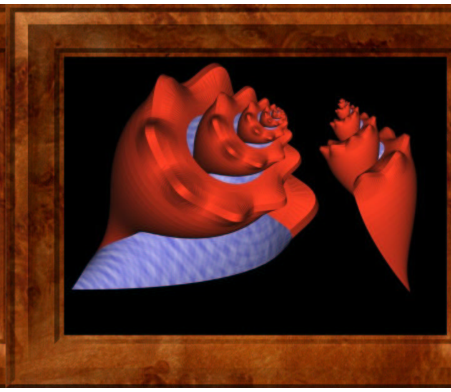

Figure 3. Lyria, 29970 in 45 sec, Can be considered a Non-Manifold Input.

Figure 4. Fist, 16384 points in 30 sec

2 The Algorithm

The basic algorithm of *Reviver* is very similar to the Minimum Spanning Tree except that the problem with the Minimum Spanning Tree in $\mathbb{R}^3$ is that it is not a surface but a set of edges which is quite sparse [2]. *Reviver* integrates the manifold extraction step with the normal filtering step, that most current theoretically provable algorithms do in separate stages. *Reviver* starts with creating the Delaunay tetrahedralization of the input point set. *Reviver* then tries to find the MST of the dual of the restricted delaunay triangulation where the verices of the dual graph are the circumcenters of the triangles of the restricted delaunay. It is an adaptation of a advancing front algorithm that also enforces that the front is always a manifold. The MST favors the front propagation to be in the direction of low curvature in the data if the sampling is somewhat uniform. It also helps to avoid ambiguous situations encountered when trying to propagate the front across sharp edges(As for example in the dataset Lyria). *Reviver* does a shrinked equitorial sphere test(*SEST*) before it starts growing the front, and constantly prunes the set of triangles as the front expands. The *SEST* checks if a diamtral sphere of a triangle or a small shrinkage of it, is empty of sample points. If not, the triangle is filtered out.

A triangle is included in the front if it has the smallest circumradius among the triangles incident on the edge of the front and if:

① The new triangle t' does not make a dihedral angle of less than $\frac{\pi}{2}$ with the traingle t it is incident on in the front.

② If in the normal direction of t, t' has a sliver incident on it, it selects the next triangle in the following manner.

- Go to the next triangle towards the normal to t which makes a sliver angle with t'. Let this triangle be t' now.
- Repeat the previous step if t' still has a sliver incident on it towards the normal to t.

③ If conditions 1 and 2 are satisfied, check that t' does not create a non-manifold.

The normals to the triangles in the front are maintained in correct orientation all the time. A Priority Queue is used to maintain the set of candidate triangles incident on the front whose weights are kept proportional to their circumradius. No special boundry or sharp edge handling is employed in *Reviver* except manifold conditions at vertices and edges.

3 Implementation

Making Qhull work for *Reviver* did take a considerable amount of time. *Reviver* uses Exact Arithmetic of LEDA when it cant say if a shrinked smallest sphere enclosing a triangle is empty of sample points or not. The walk on triangles filtered from the Delaunay triangulation is simple advancing front implementation with a priority queue on the edges of the front. *Reviver* can only provably reconstruct smooth closed surfaces, but it does work well on point sets with borders and sharp edges when the turn is not an acute angle. It cannot reconstruct non-manifolds and manifolds whose reconstructions contains an edge that has two triangles incident on it, whose normals make an angle of more than $\frac{\pi}{2}$ in proper orientation.

References

[1] Nina Amenta and Marshall Bern. Surface Reconstruction by Voronoi Filtering. In *Proc. 14th Annual ACM Symp. on Computational Geometry* (1998) 39-48

[2] Hugues Hoppe, Tony DeRose, Tom Duchamp, John McDonald and Werner Stuetzle. Surface reconstruction from unorganized points; *Proceedings of the 19th annual conference on Computer graphics*, Pages 71 - 78 (1992)

[3] Herbert Edelsbrunner and Ernst P.Mcke. Three-dimensional alpha shapes. Software Available at ftp://ftp.ncsa.uiuc.edu/Visualization/Alpha-shape/, *ACM Trans. Graph.* **13**, 1 Jan, Pages 43 - 72 (1994)